Bridging Socioeconomics and Ergodicity Economics: Power Laws, Multiplicative Processes, and Persistent Stratification

Ergodicity Economics Conference 2026

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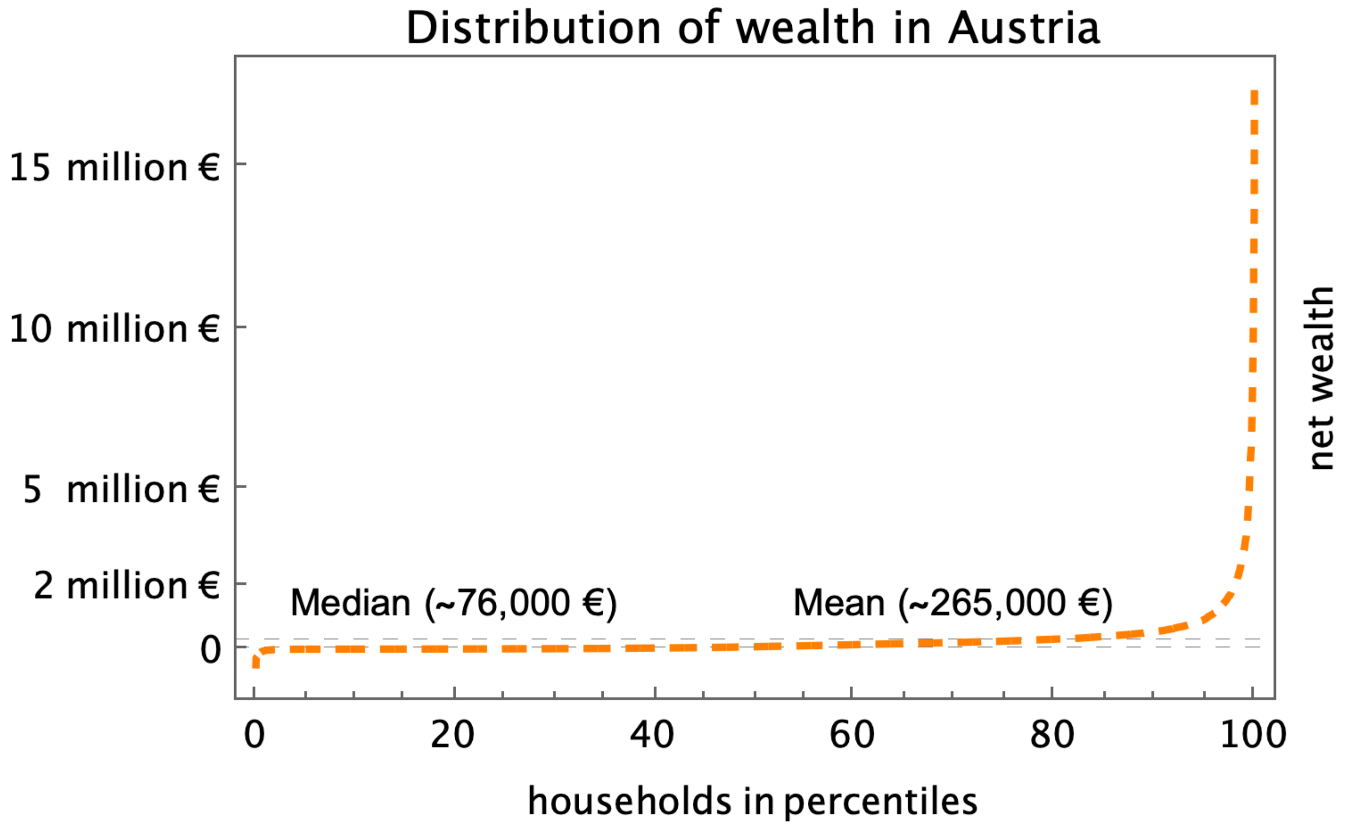
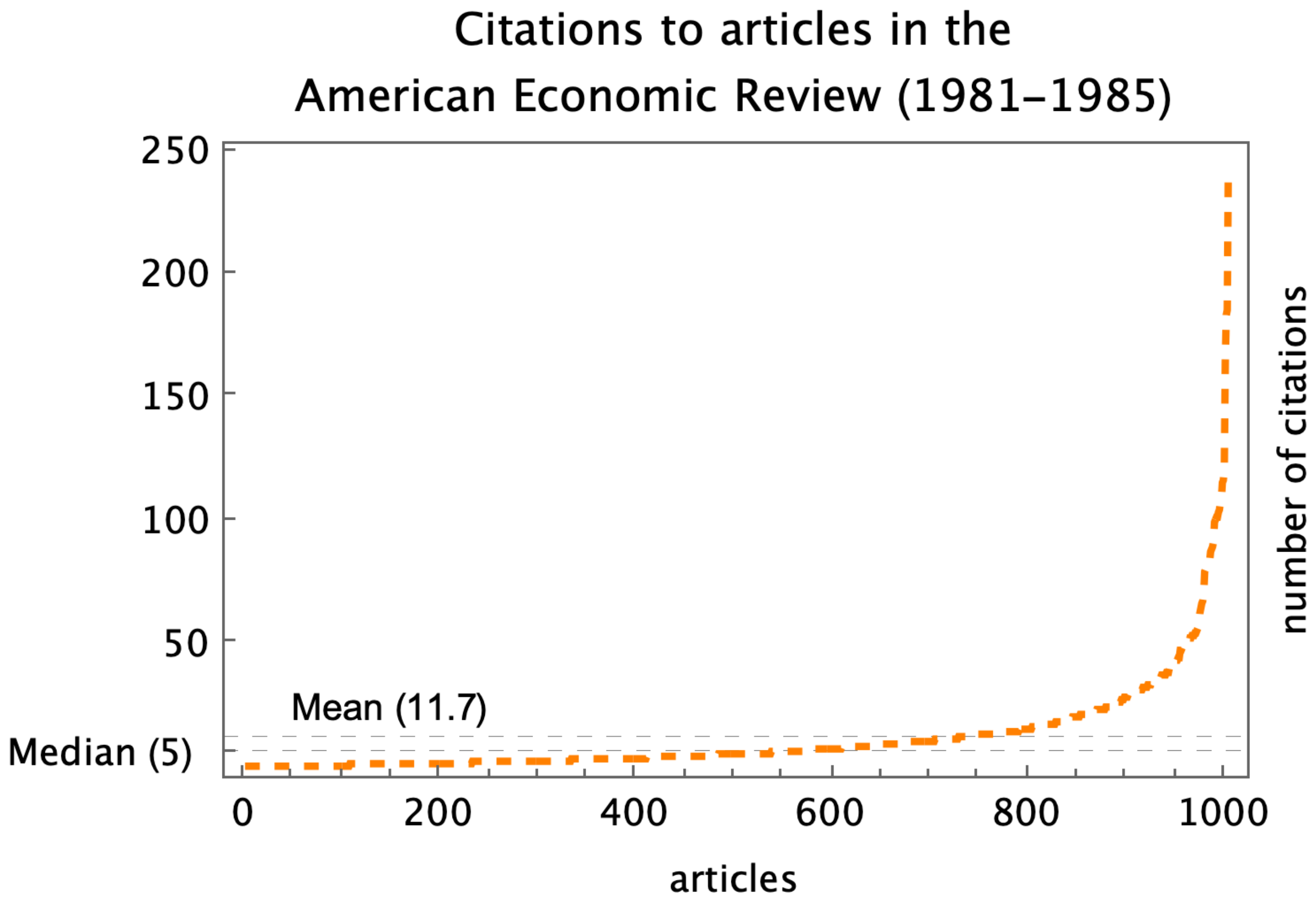
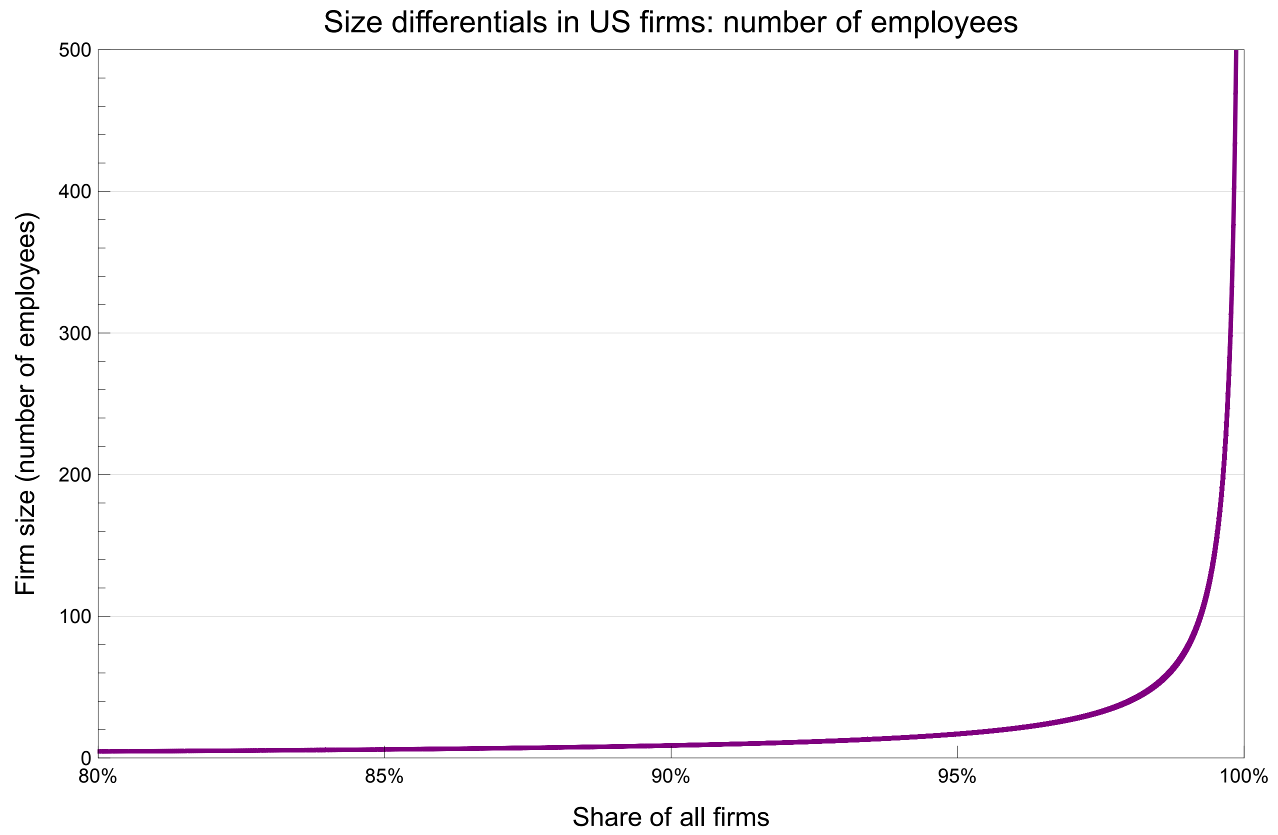
Power laws in socioeconomics: a non-ergodic (💖-)story?

- Power law distributions repeatedly pop up in our research: wealth and income distribution, citations, firm size, market power (e.g. technology adoption, profit rates), visibility/power in social networks etc.







- Heterodox traditions already emphasize path dependence (e.g. Kaldor, Myrdal, David, Arthur), stability-issues (e.g. Pasinetti equation: $r \cdot s_\pi = g$), Matthew effects (e.g. $s_\pi > s_w$ or $r_{rich} > r_{poor}$ …)
- The ergodicity connection has rarely been made explicit in heterodox economics / socioeconomics

➡ EE provides the formal language, socioeconomics a rich empirical testing ground and theoretical intuition

Power laws, generative mechanisms and socio-economics

- Power law distributions: regular aggregate property of socio-economic variables
  - But what happens in terms of actual processes, that drives this (regular) pattern?

- Newman (2005): Power laws, Pareto distributions and Zipf's law, *Cont. Physics*
  - (1) Combinations of exponentials → Mainstream!
  - (2) Taking inverses of quantities
  - (3) Random walks
  - (4) The Yule process (and other ‚rich-get-richer' mechanisms)
  - (5) Phase transitions and critical phenomena
  - (6) Self-organized criticality
  - (7) Other mechanism including, „multiplying together random numbers"

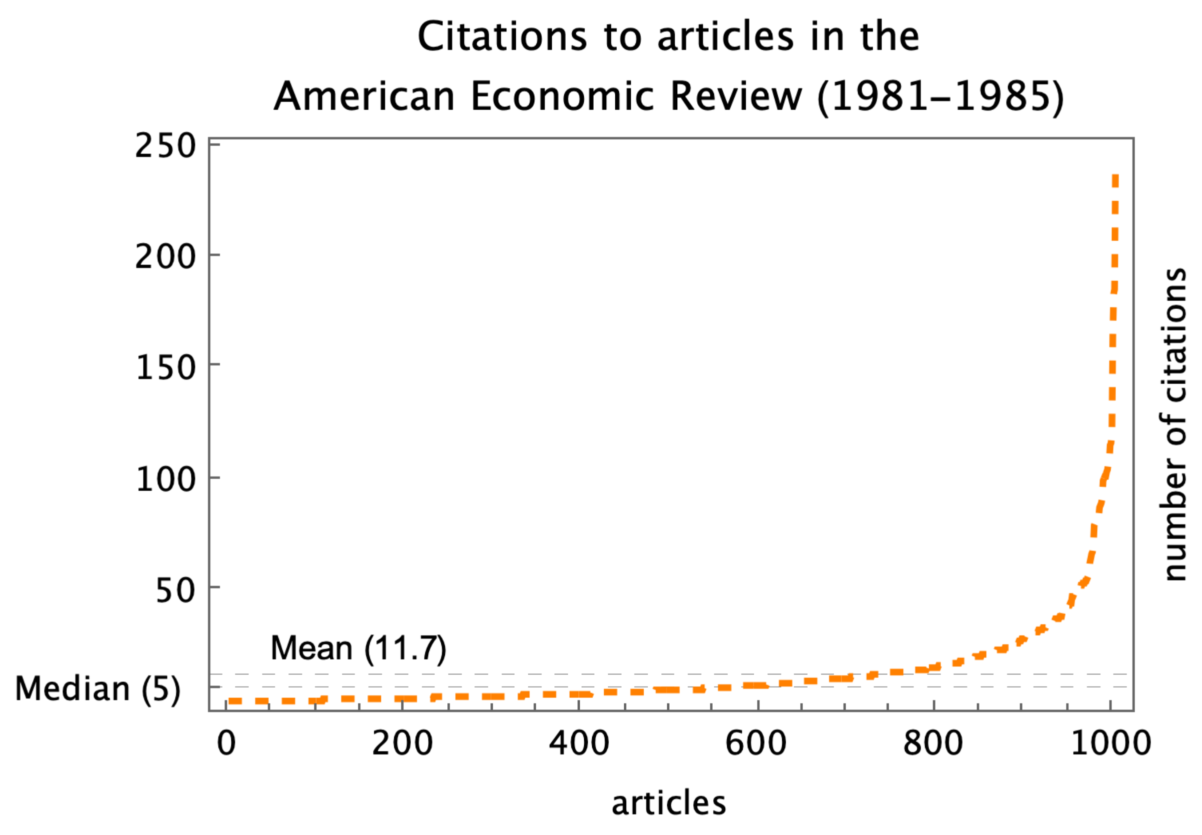
$\mathbb{P}(Talent \geq x_i) = e^{-\delta x_i}$ and $w_i = e^{\lambda x_i}$

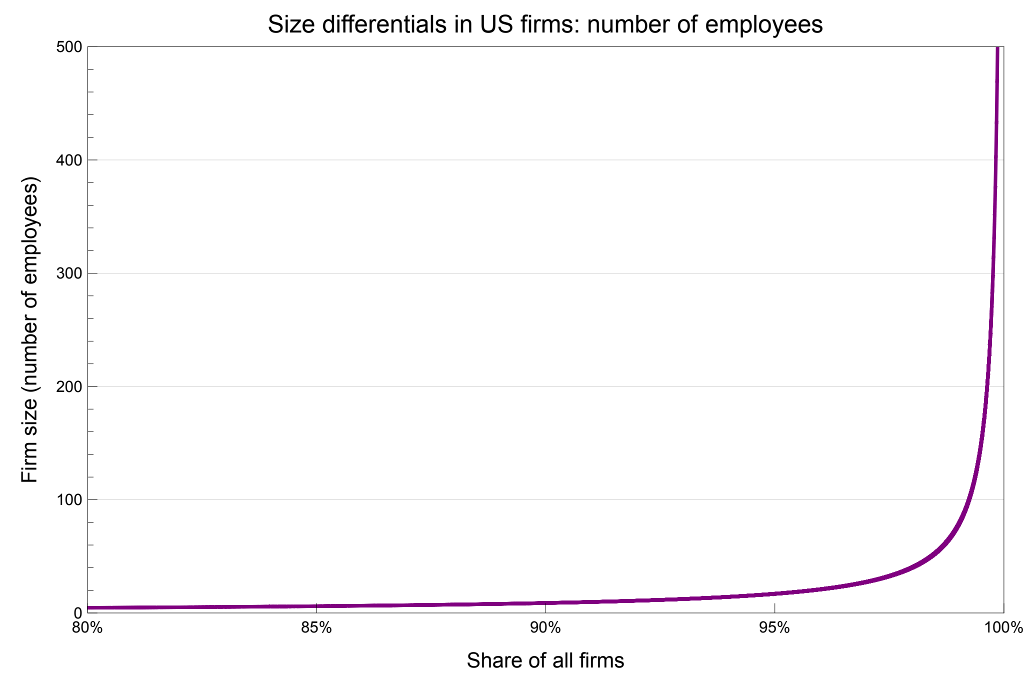
$x_i = \frac{log(w_i)}{\lambda}$

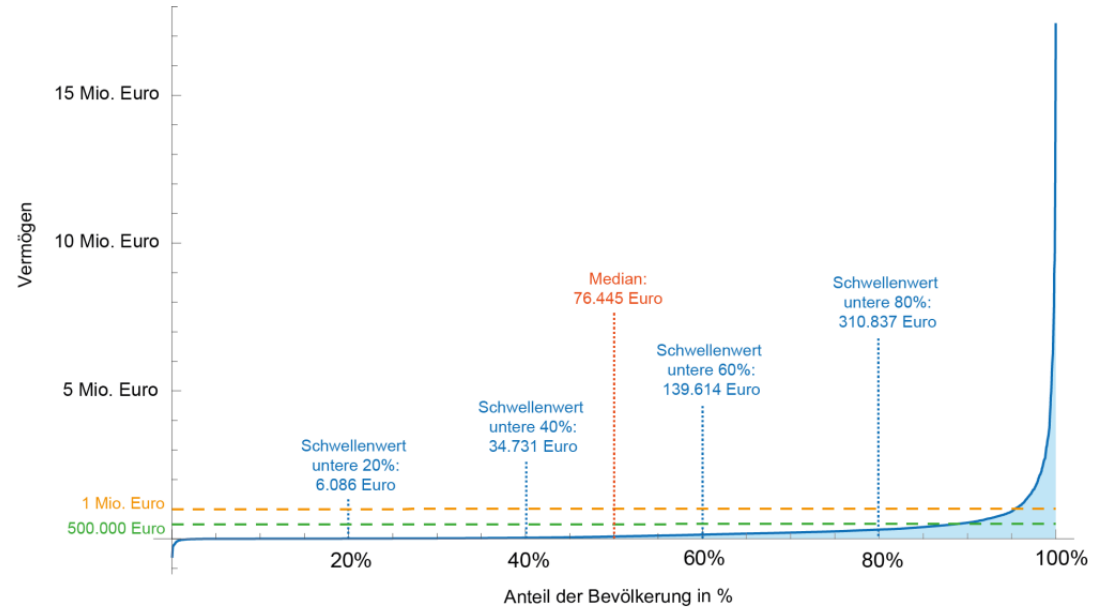
$\mathbb{P}(wealth \geq w_i) = w_i^{-\frac{\delta}{\lambda}}$

These two can be found in finance!

Inequality is stable & steady-state exists!







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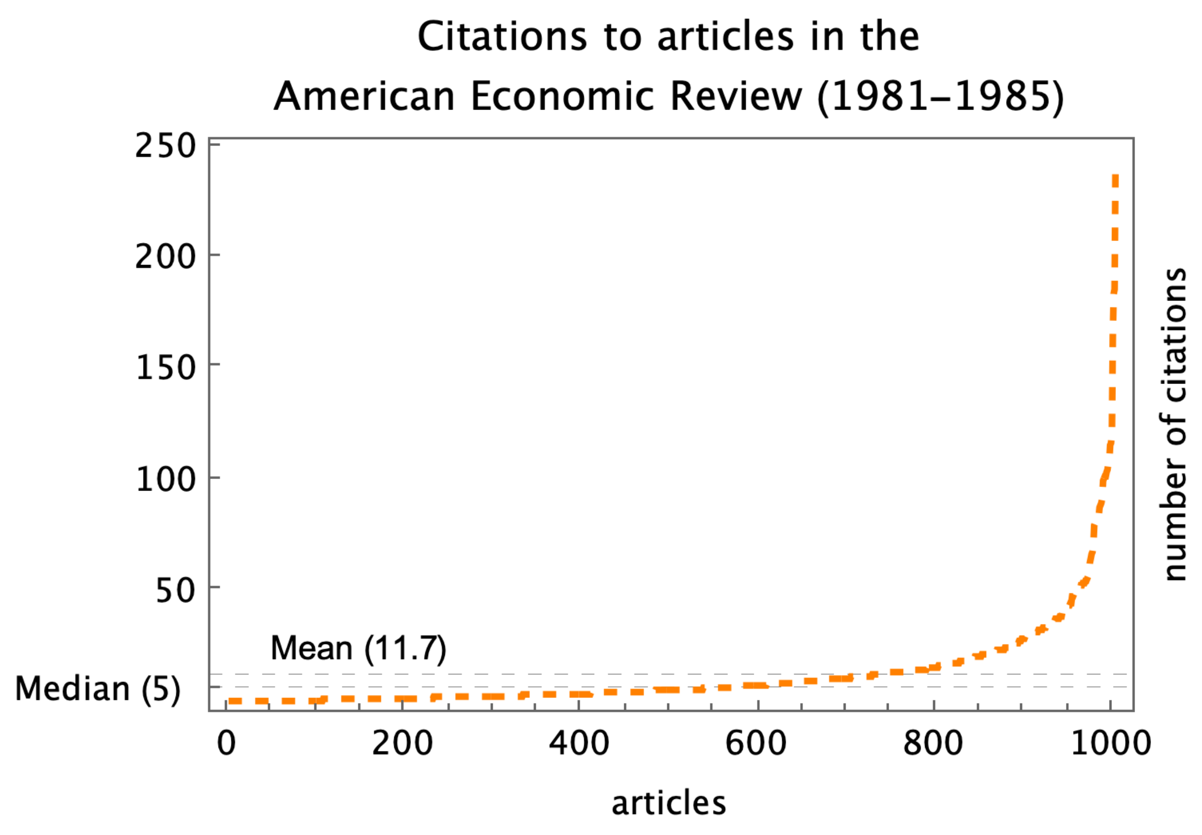
$$x_i = \frac{log(w_i)}{\lambda}$$

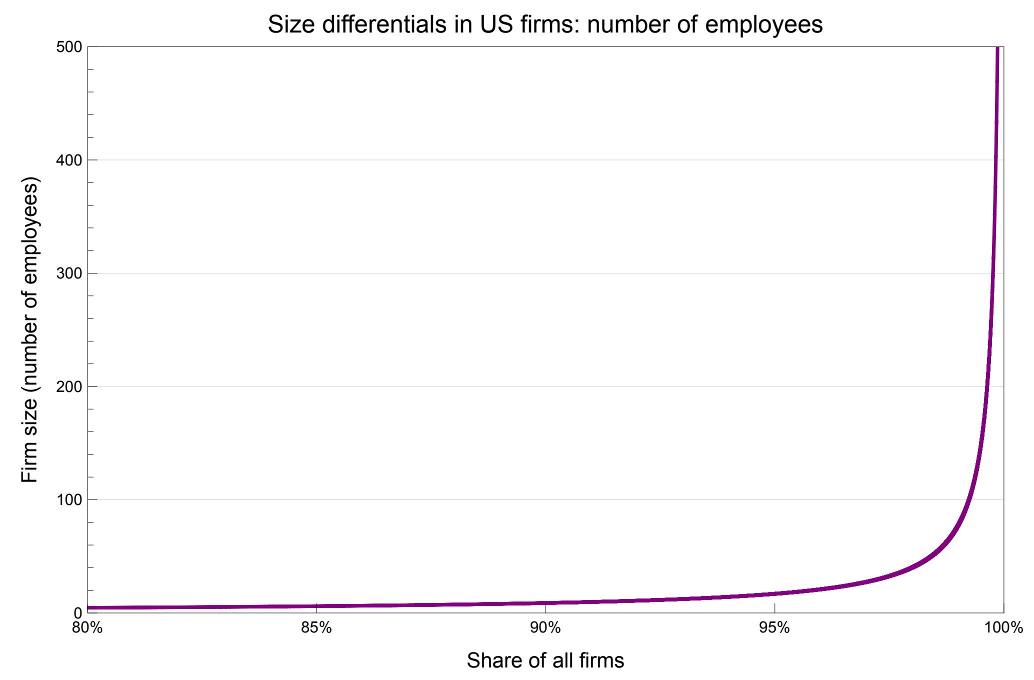
$$\mathbb{P}(wealth \geq w_i) = w_i^{-\frac{\delta}{\lambda}}$$

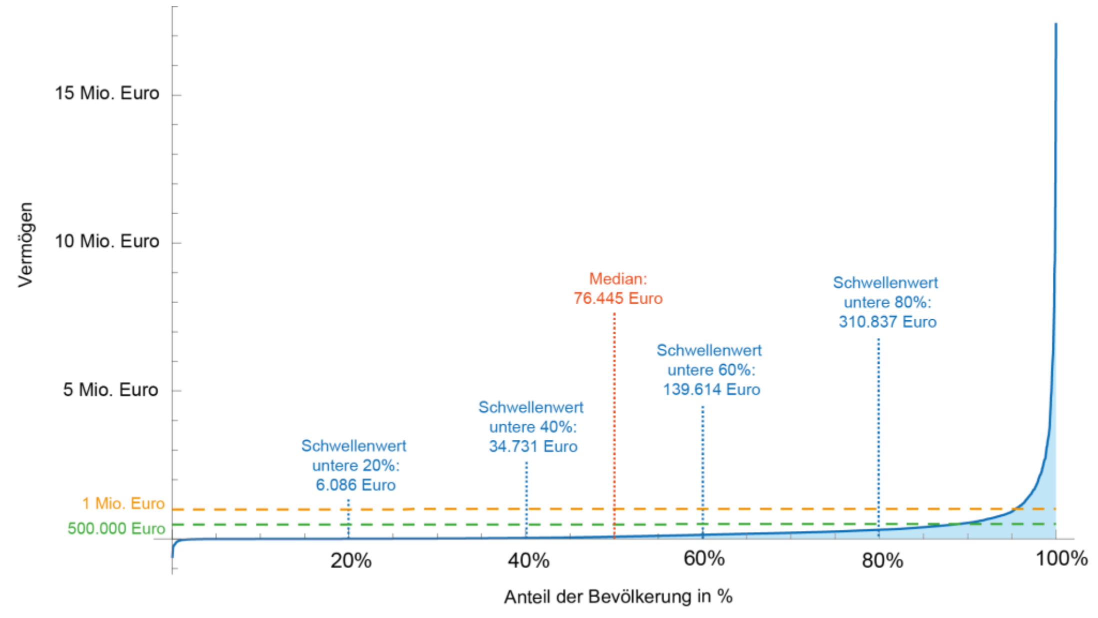
→ Inequality is stable & steady-state exists!

(4) → Simple models of cumulative advantage → Inequality can easily be exploding → no steady-state exists!

(7) → Random multiplicative growth: „Gibrat model" → Inequality can easily be exploding → no steady-state exists!





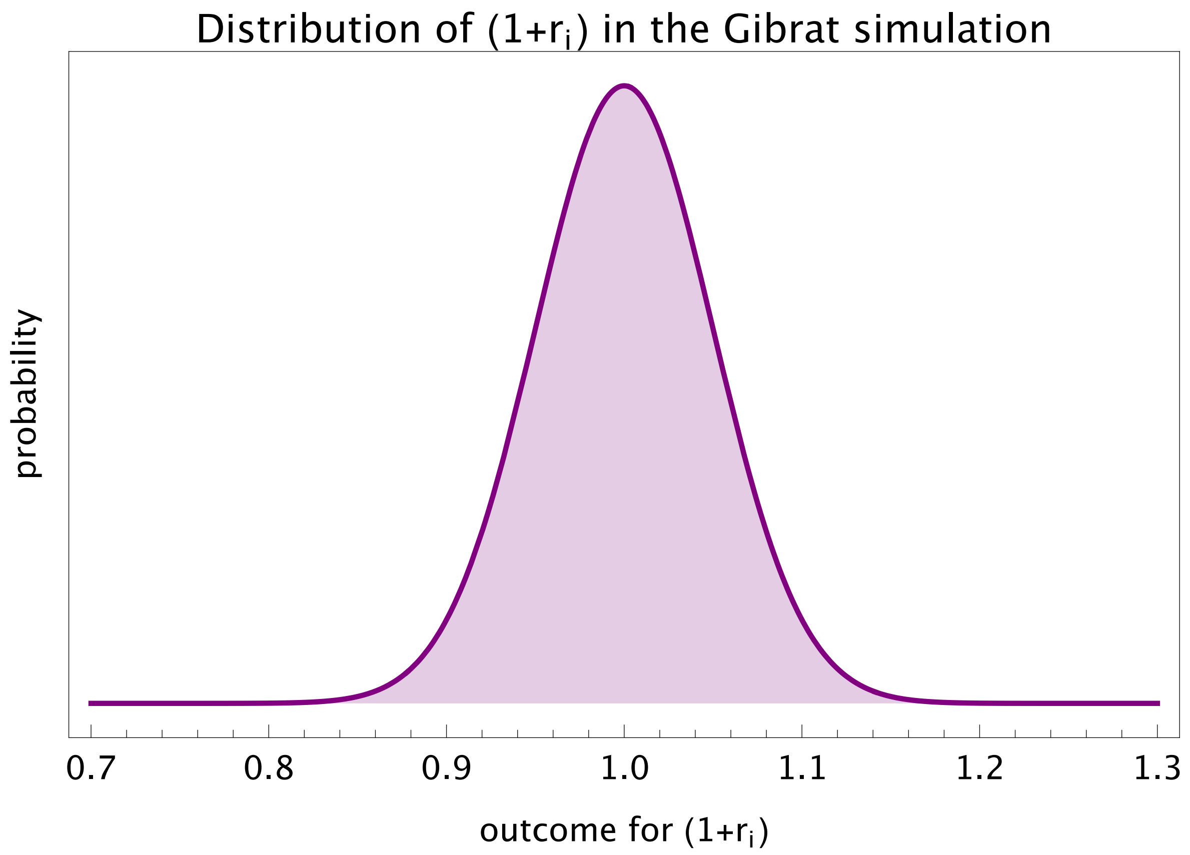


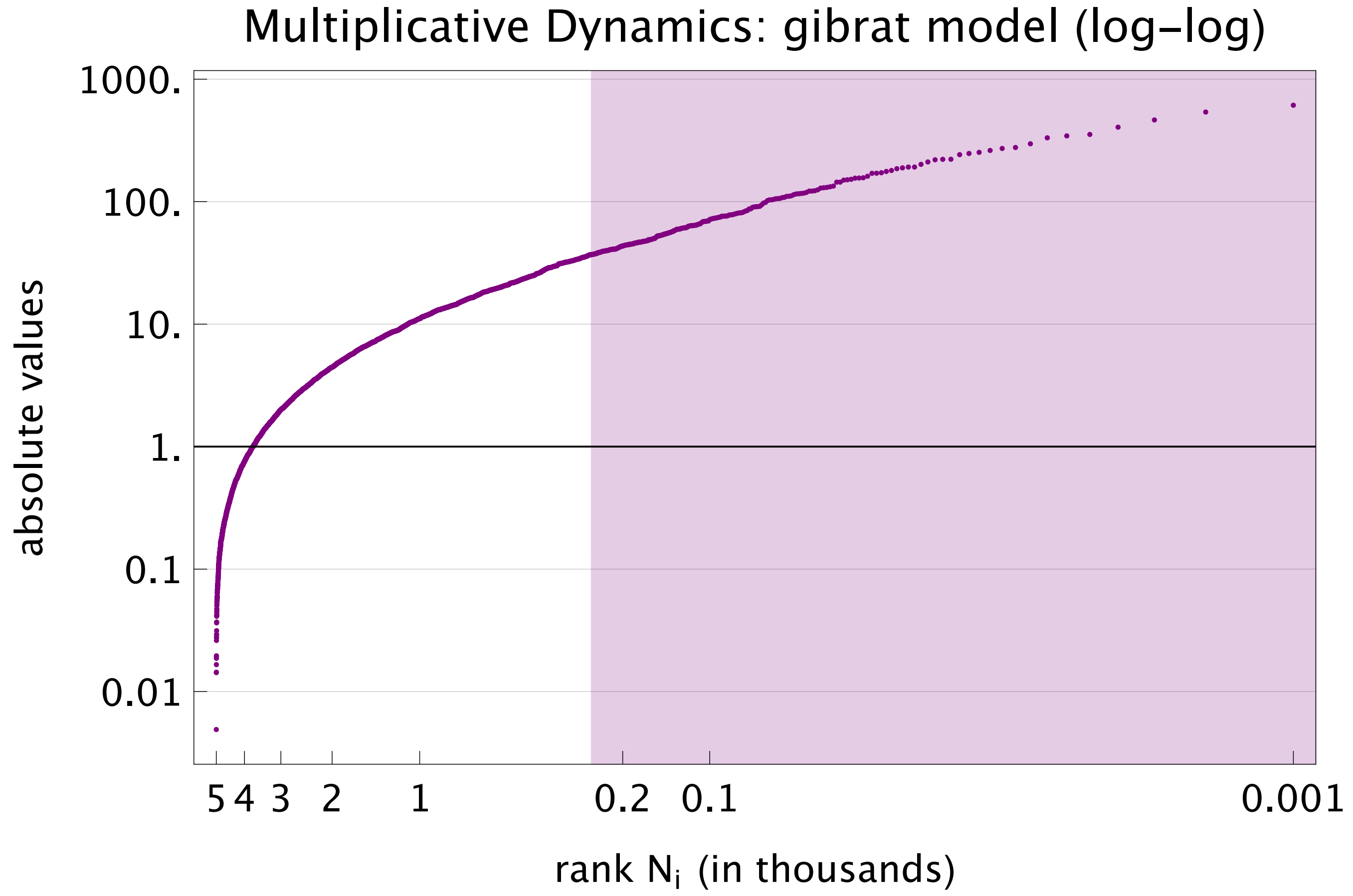
Models of random multiplicative growth as a shared vantage point

- The classic „Gibrat model" looks like…

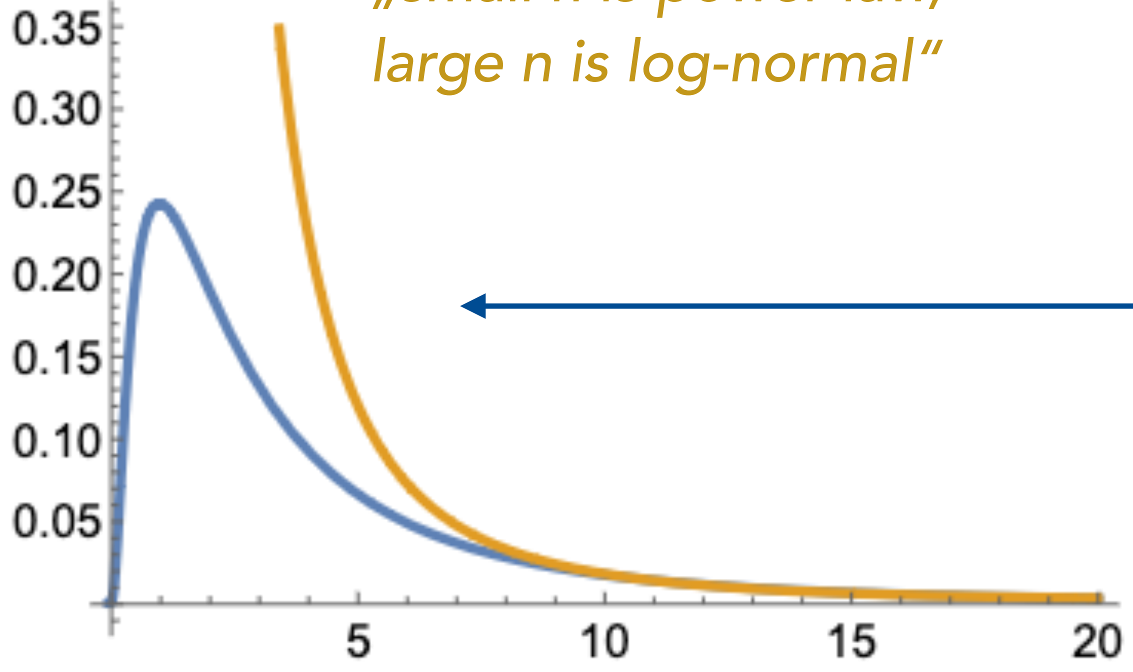
$$w_{i,t} = (1 + r_{i,t}) \cdot w_{i,(t-1)} \qquad \text{with} \qquad r_{i,t} \sim \mathcal{N}\left(\mu, \sigma_G\right)$$

  - Well-known as a sort of ‚null-model' explaining the emergence of power laws in socio-economics.
  - Crucial: $E(r_i) = E(r_j)$, no strict bifurcation
  - Long-term outcome: $\log(w_T) \sim \mathcal{N}(t \cdot \mu, t \cdot \sigma_G^2)$

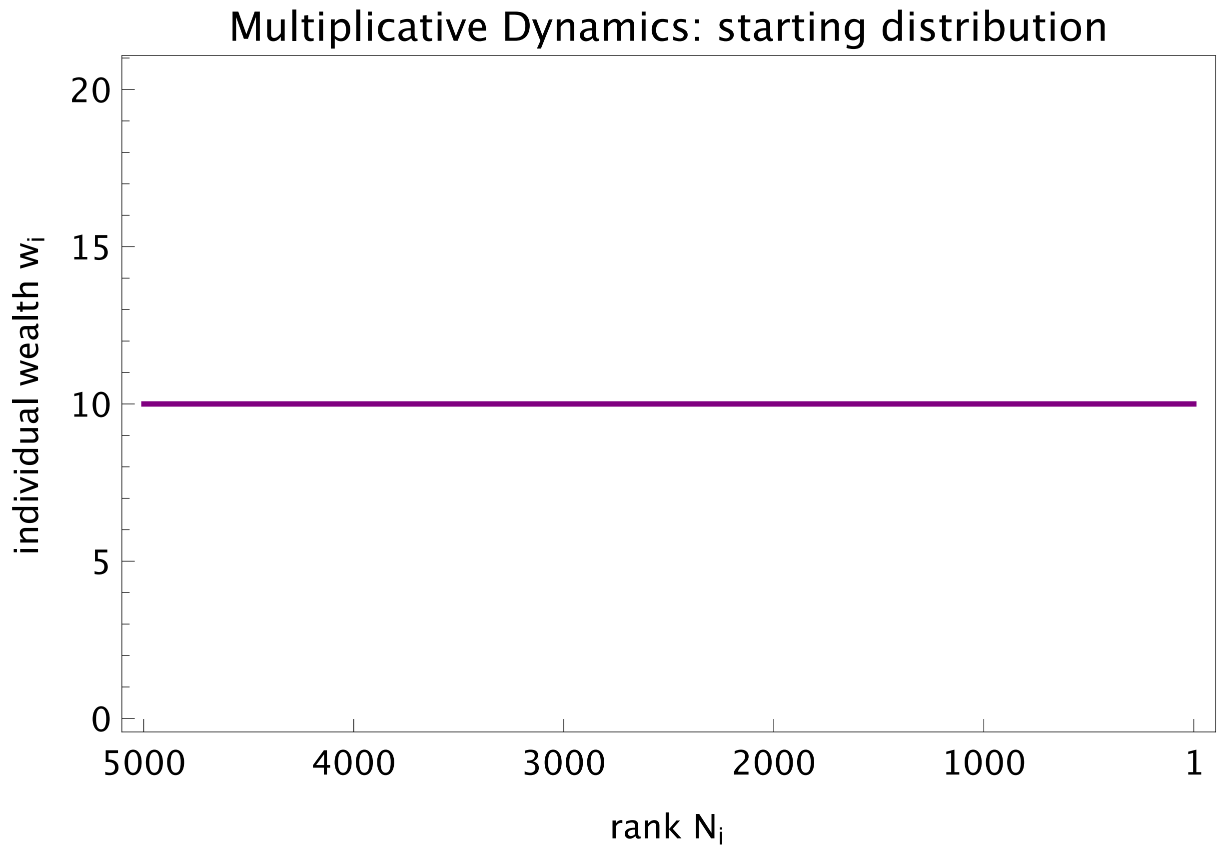


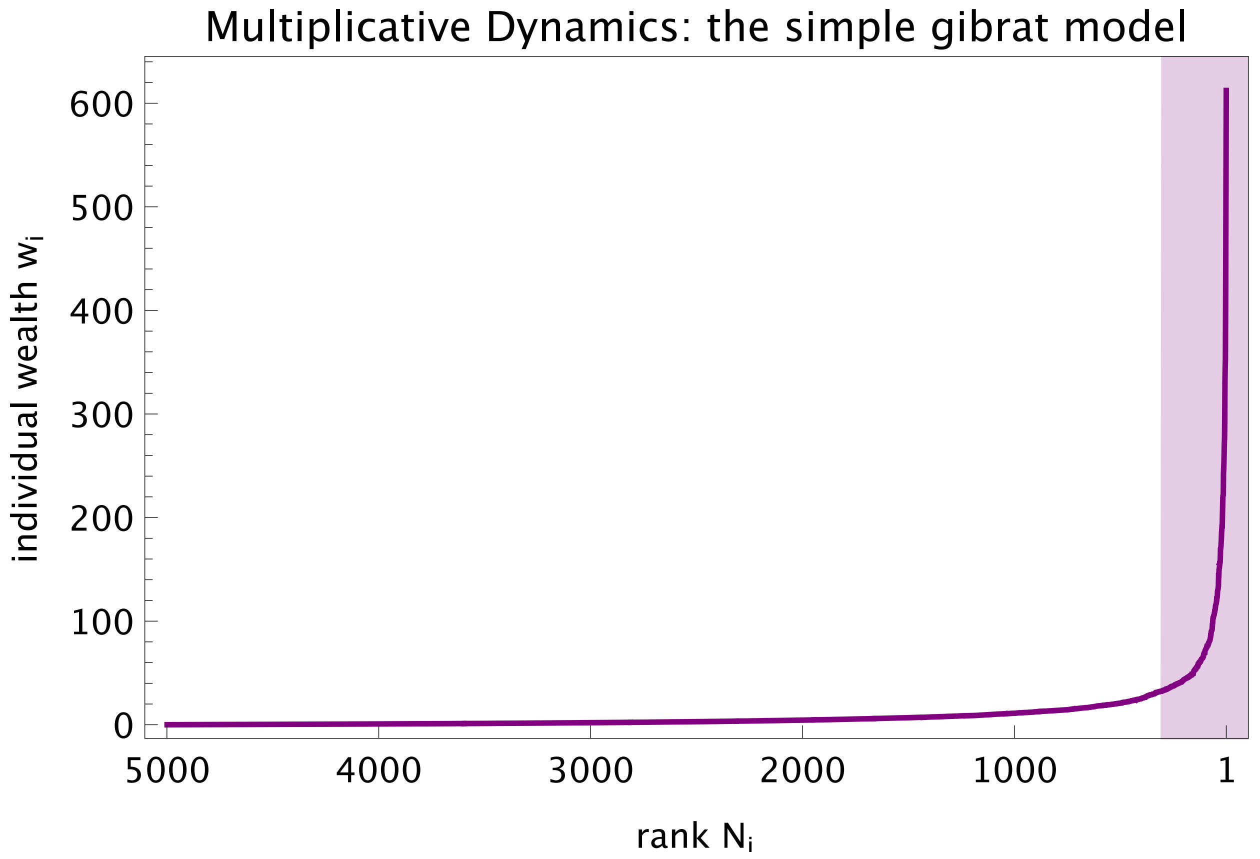


Mandelbrot-quip:
„small n is power law,
large n is log-normal"



Wealth follows a
log-normal distribution
In this model!





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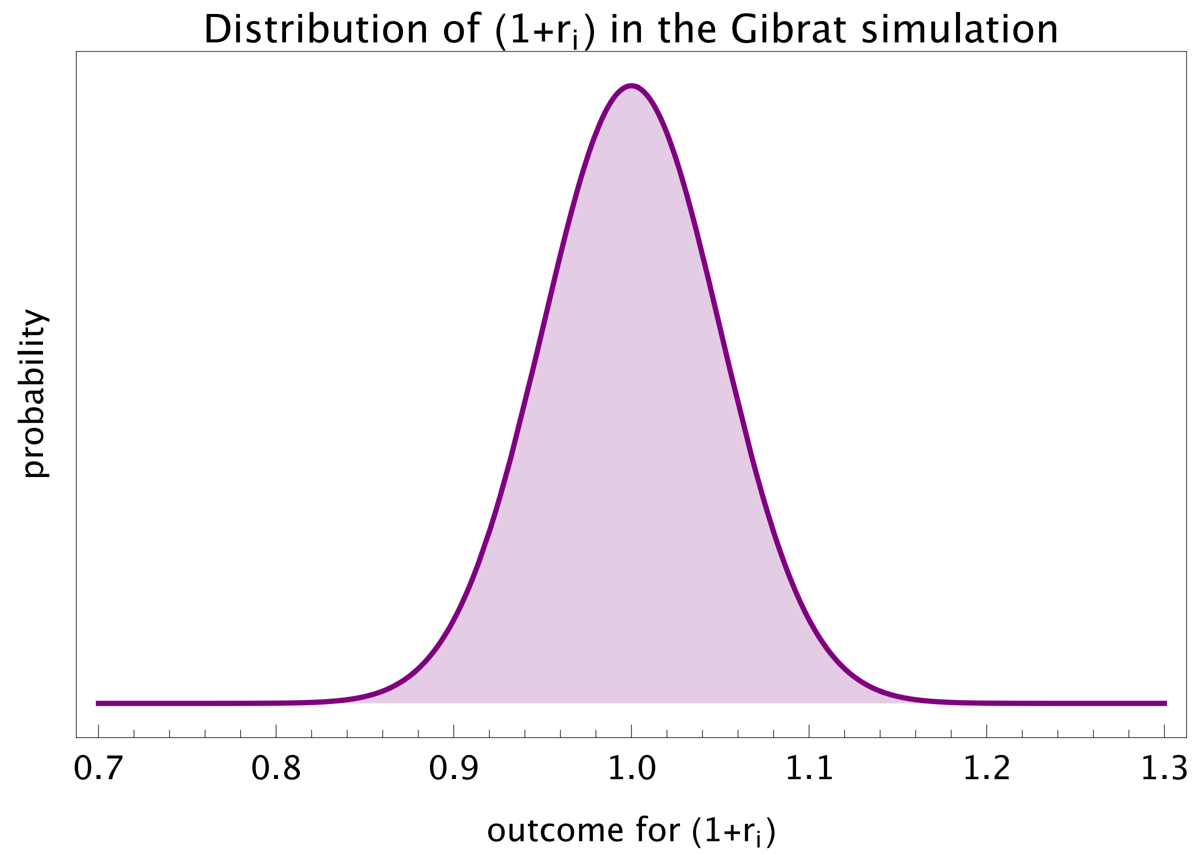
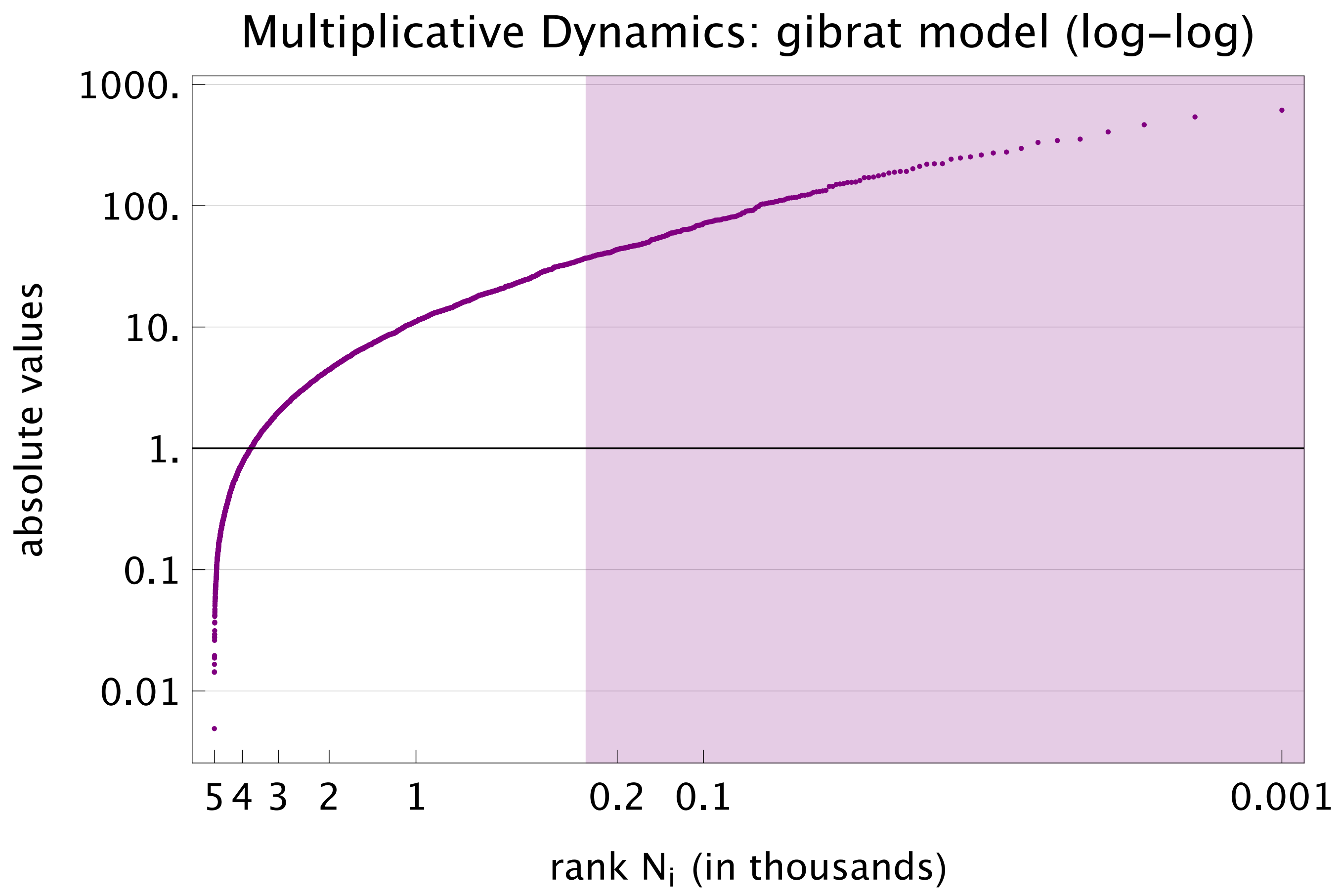
Exemplary socio-economic follow up questions?

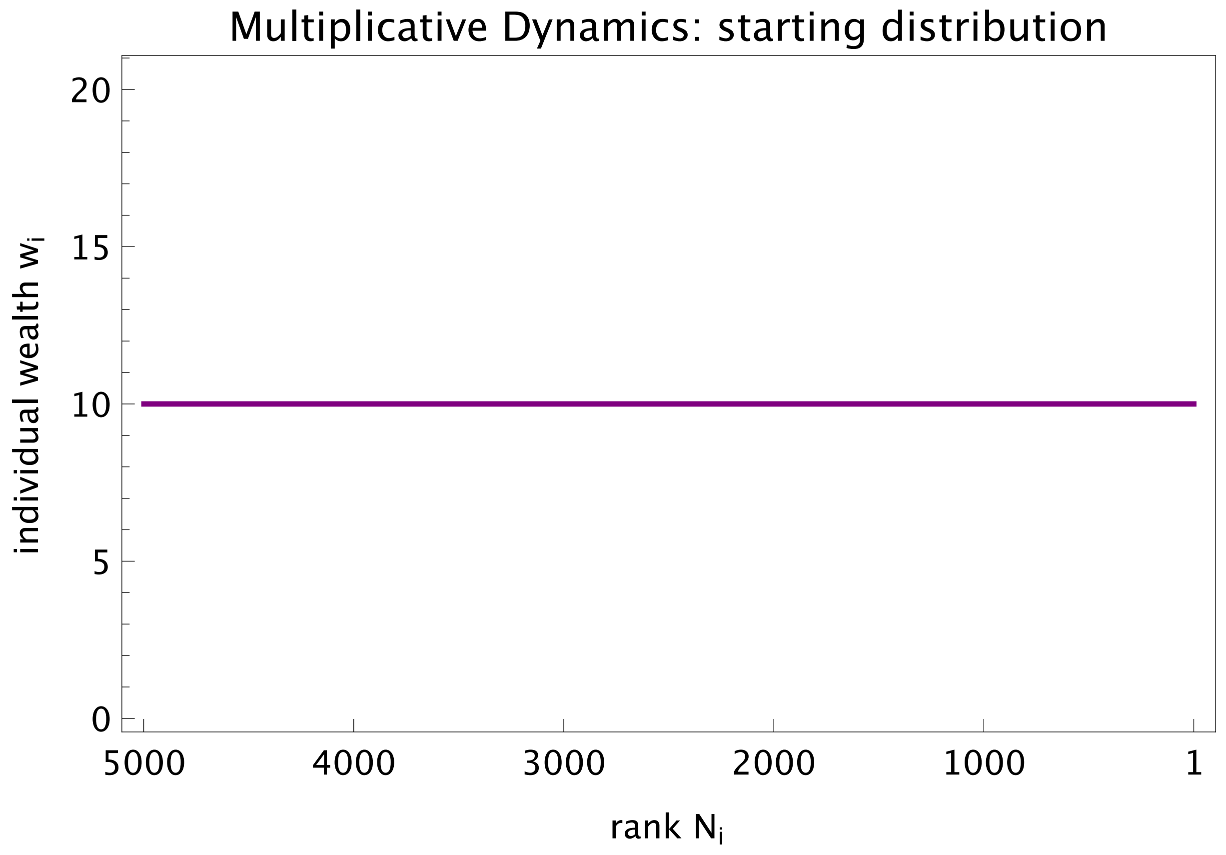
(1) What is the role for path-dependence in this setup?

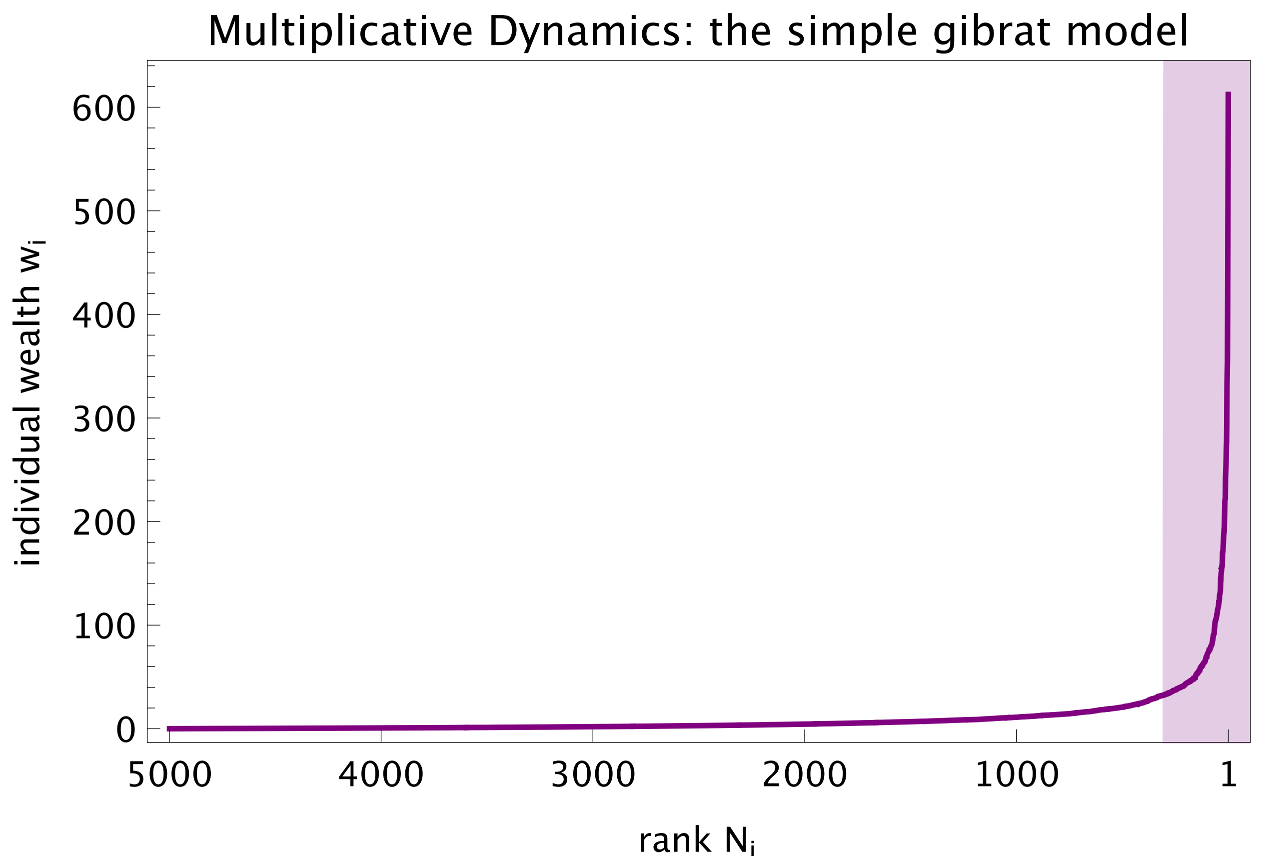
(2) How does this map on historical data?

(3) Are there complementary generative mechanisms that are relevant from a socio-economic perspective?









Question (1): Path-Dependence

Models of random multiplicative growth and path-dependence?

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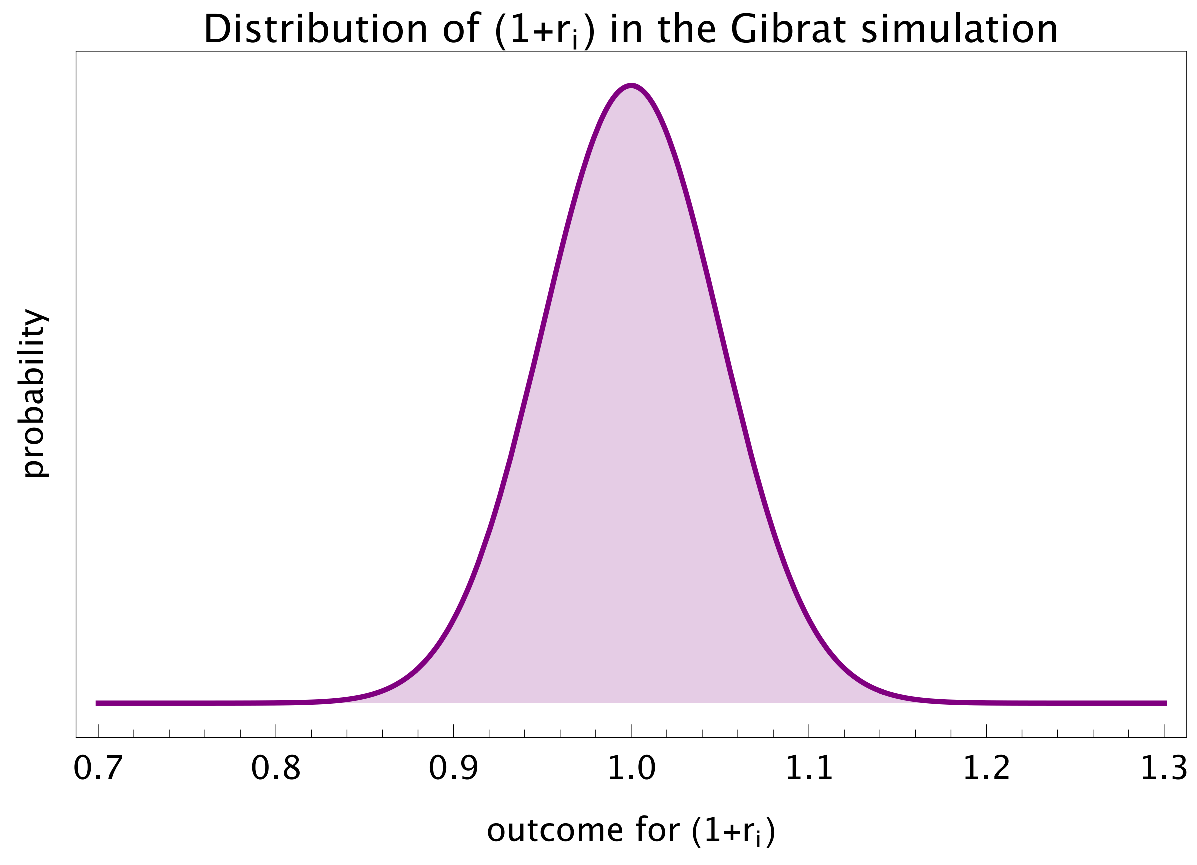
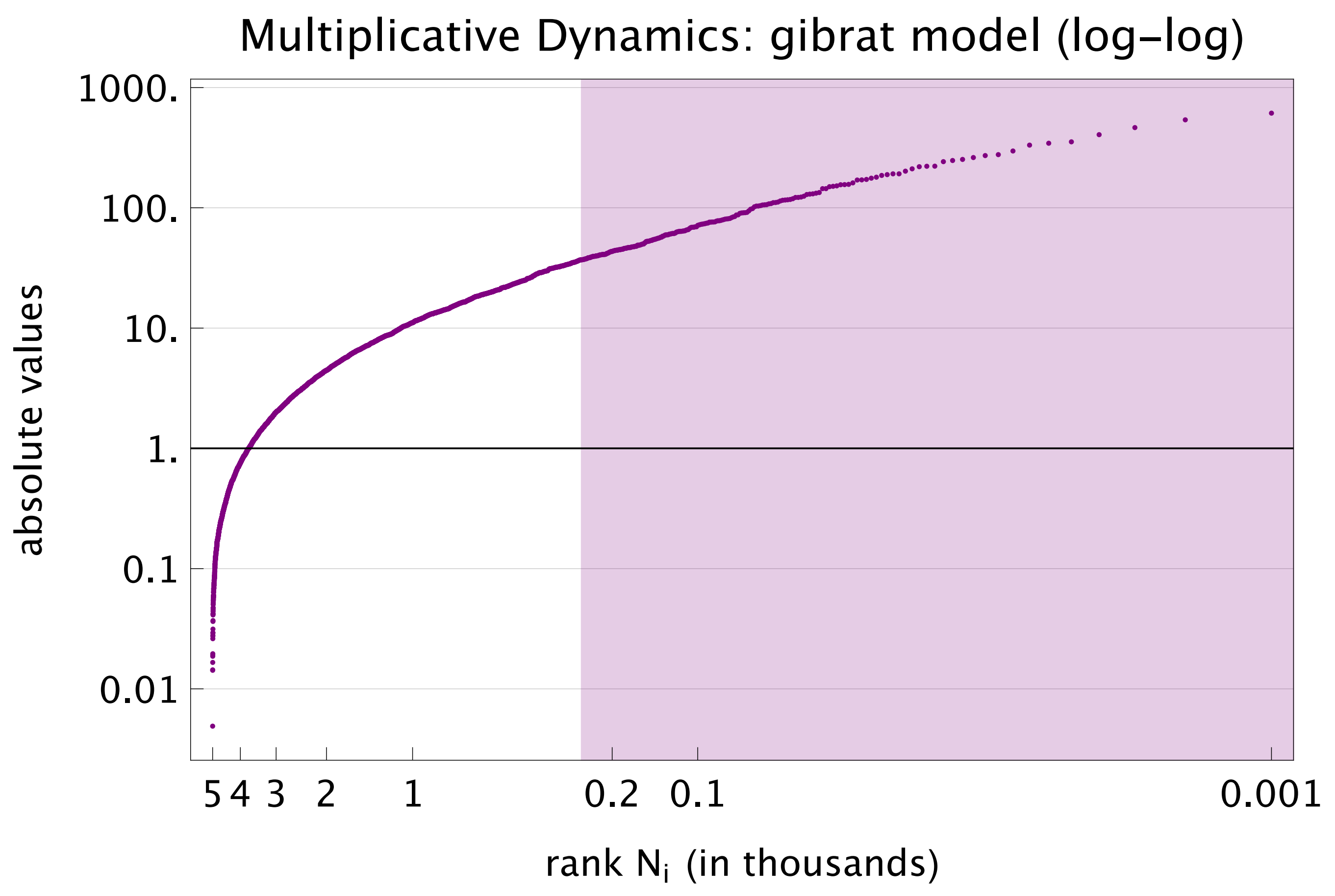




  - Well-known as a sort of ‚null-model' explaining the emergence of power laws in socio-economics.
  - Here we have a multiplicative random walk: in eternity, everybody will rise & fall – are there plausible venues to align this with the heterodox intuition of path-dependence and persistent asymmetry?

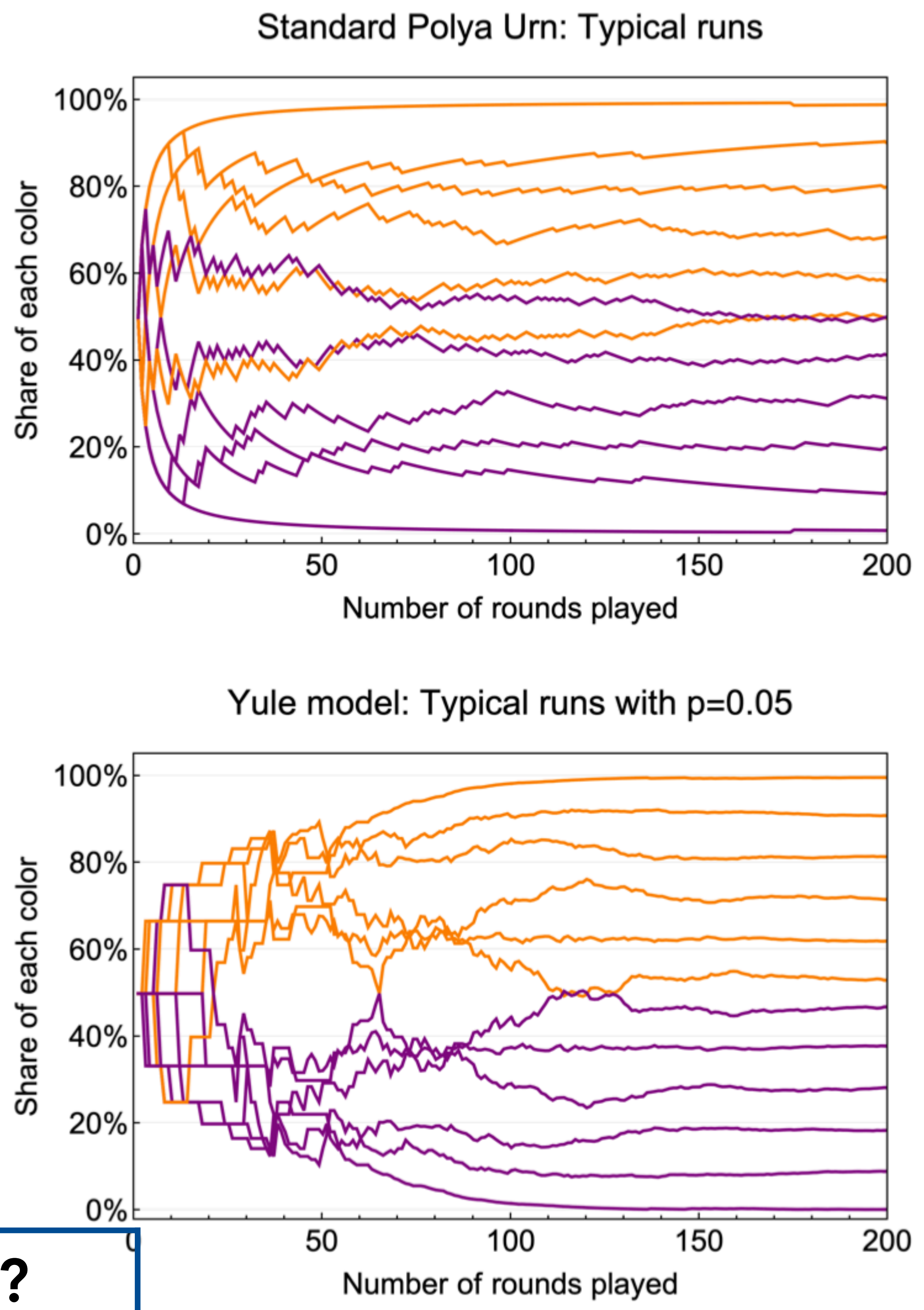
- Random multiplication and path-dependence: three modest venues
  - (1) The „leading player's advantage": if A is richer than B, the p that A stays richer in all future periods is $p > 0.5$.
  - (2) We can think about time-scales! If a participant is born in round 800 and lives for 60 periods, the world will be super-path-dependent → quasi-non-ergodic dynamics
  - (3) We can introduce minimal variations to achieve persistent asymmetry without changing the key assumption that $E(r_i) = E(r_j)$.

Question (1): Path-Dependence

Models of random multiplicative growth and path-dependence?

- Heterodox economics has formal models for persistent advantage:
  - Pólya urn: balls of different colors grow if drawn and probability of drawing same color again spends on share – early random leads become self-reinforcing
  - Yule model (simplified): each unit of each species reproduces independently with constant probability p — large stocks have stable r, small stocks very volatile r
- Both models are variants of the same multiplicative growth framework:

$$w_{i,t} = (1 + r_{i,t}) \cdot w_{i,t-1} \qquad \text{with} \qquad \begin{cases} r_{i,t} \sim \mathcal{N}(\mu, \sigma) \\ \mathbb{E}(r_{i,t}) = \mathbb{E}(r_{j,t}) \ \forall\, i, j. \end{cases}$$



Model	E(r)	Var(r) / E(r)	path-dependence?
Gibrat / GBM	constant	constant	weak asymmetry
Pólya urn	decreasing with stock	constant	persistent asymmetry
Yule process	constant	decreasing with stock	persistent asymmetry
Cumulative advantage	*increasing with stock*	—	exploding asymmetry

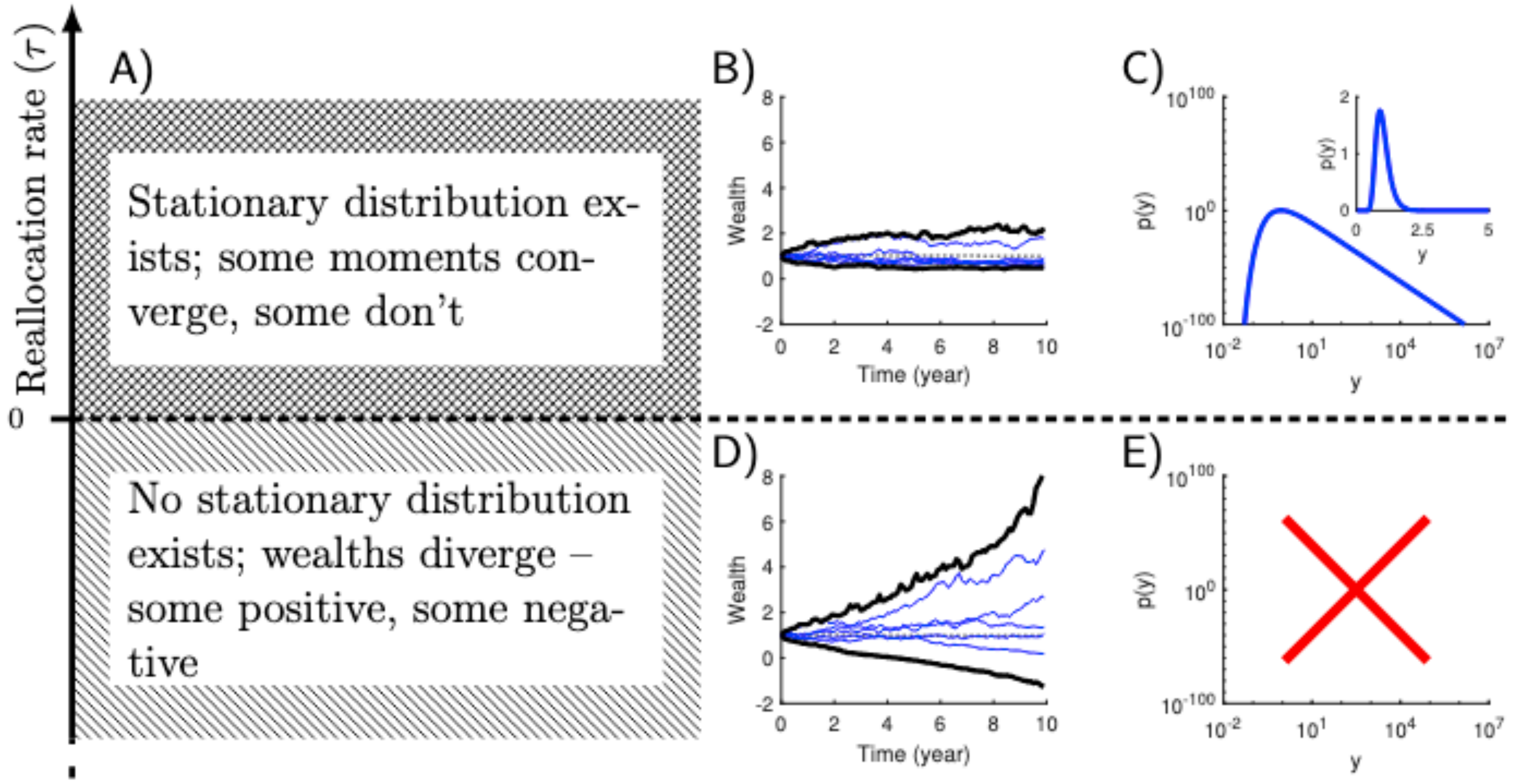
Question (2): Historical data

RMG at the Core of Income & Wealth Dynamics

- RMG is also the core building block for income & wealth dynamics in (mainstream) economics (Champernowne 1953; Benhabib & Bisin 2018)
- But RMG is inherently non-ergodic → tension with steady-state focused mainstream
- Ad hoc fix: stabilizing forces (mortality, income floors, inverse Matthew effects) → ergodicity by assumption, not by evidence.

- Can we just leave it open and test?
  → Yes: Berman, Peters & Adamou (2021)
- Reallocating Geometric Brownian Motion (RGBM):

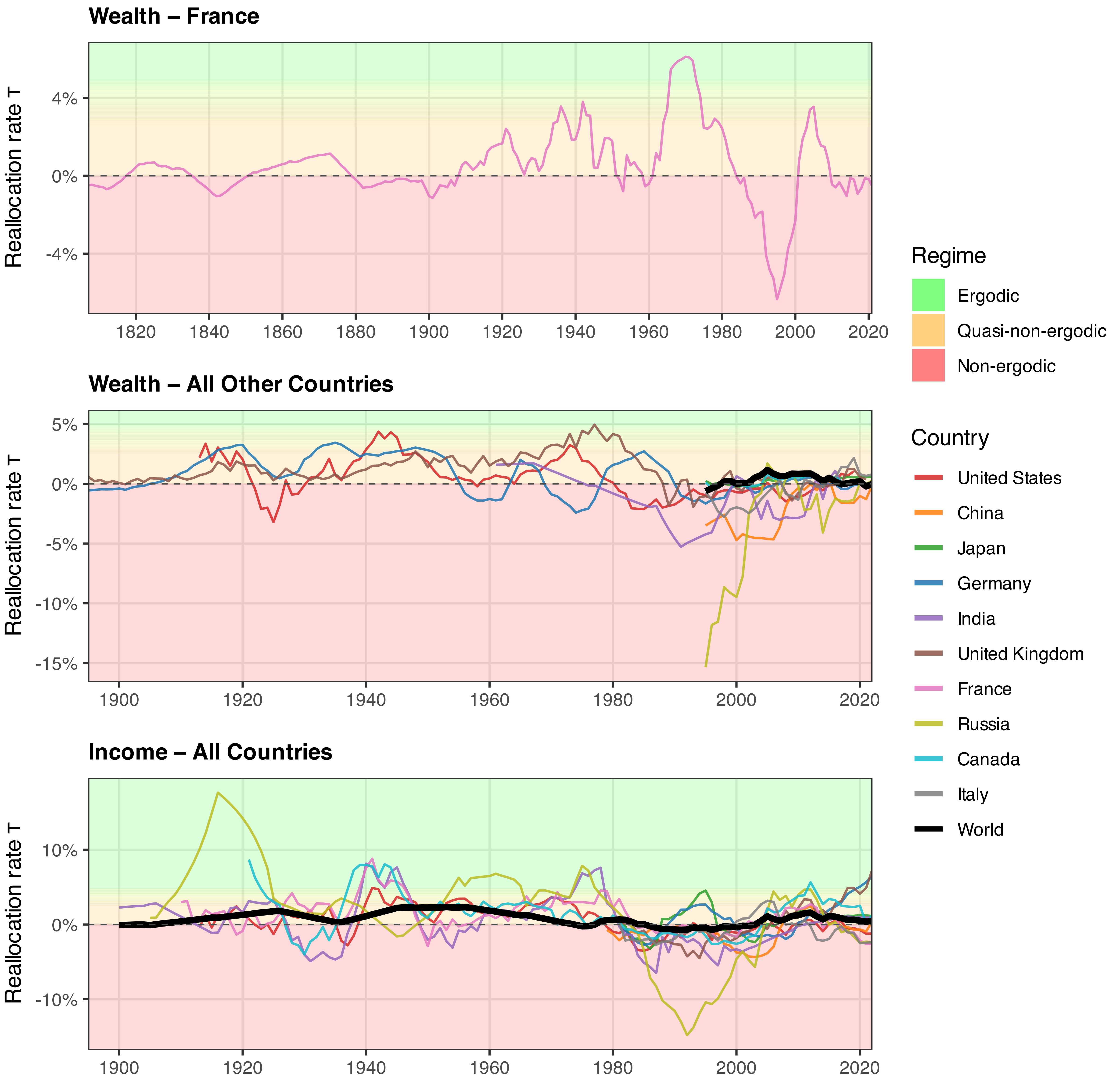
$$dx_i = \underbrace{x_i[\mu dt + \sigma dW_i(t)]}_{GBM/Gibrat} - \underbrace{\tau(x_i - \langle x \rangle_N)dt}_{Reallocation}$$



Question (2): Historical data

Extrapolating the approach to EconHistory

- Extrapolating the empirics…
  - Not just the postwar period – historical series back to the 19th century
  - Not just the US – 10 largest economies + global distribution
  - Not just wealth – income shows the same pattern
- Conclusions about „capitalism“ ;-)
  - No economy shows consistently ergodic income or wealth dynamics (~40% of cases non-ergodic)
  - Where ergodic: convergence times decades to centuries → quasi-non-ergodic in practice
  - Pronounced intensification since the 1980s



Lines show the 10-year centered moving average ($\bar{\tau}$). Background shading indicates the ergodicity regime: non-ergodic ($\bar{\tau} < 0$), quasi-non-ergodic (0–5%), and ergodic ($\bar{\tau} > 5\%$).

Question (3): Complementary mechanisms

Matthew Effects: A First Candidate

$$E(r_{rich}) \geq E(r_{poor})$$

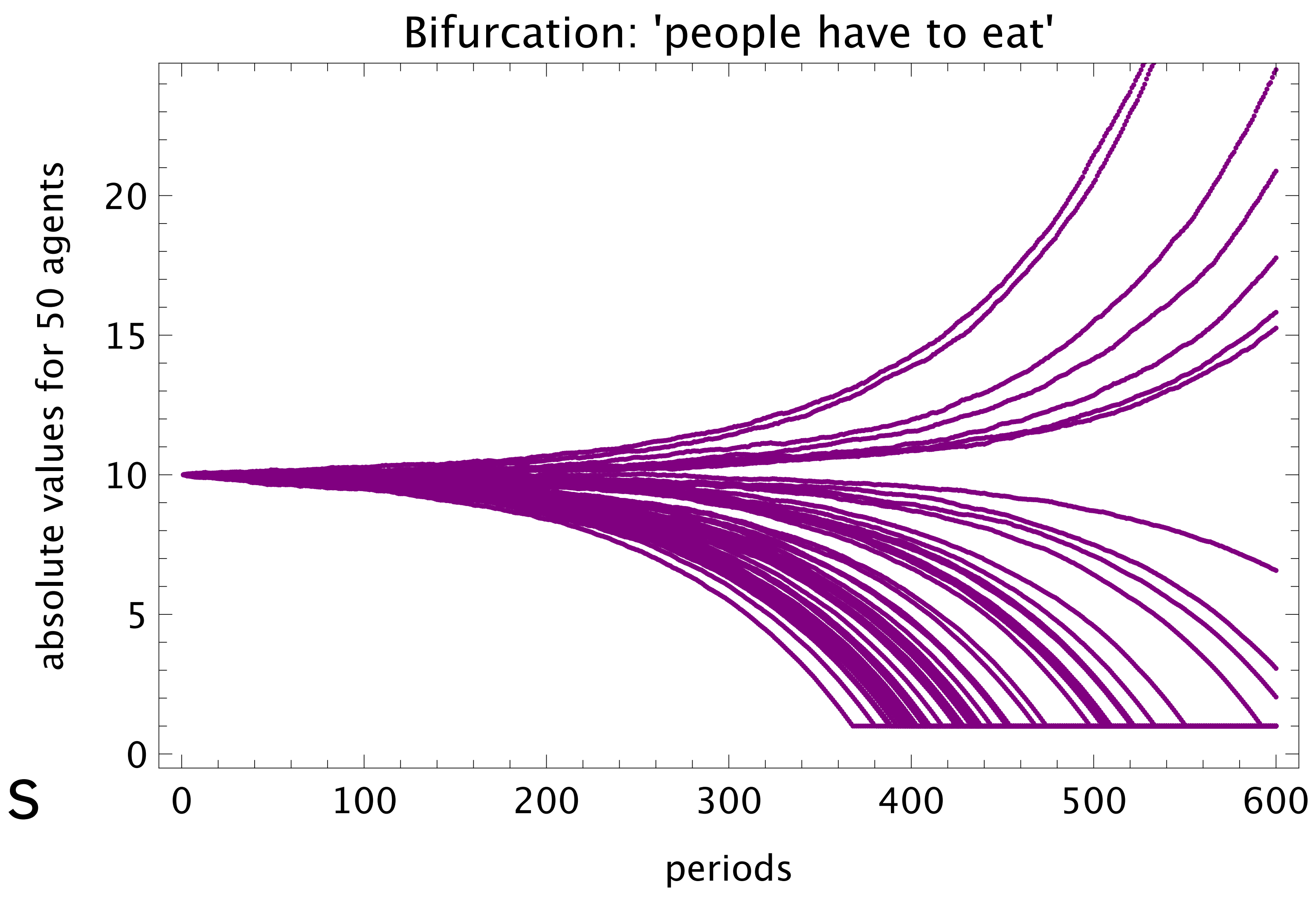
> "For whoever has will be given more, and they will have an abundance. Whoever does not have, even what they have will be taken from them.
>
> Matthew 25:29

- Central concept in economics for explaining persistent inequality
  - **Conceptual arguments:** Differential saving rates (Kaldor 1955, Pasinetti 1962), cumulative causation (Myrdal, Kaldor), tendency towards Monopoly (Smith, Marx), differential rates of return, Piketty's r > g, platform power…
  - **Models:** Replicator dynamics, technology adoption with positive feedback

    Simple example:

$$w_{i,t} = (1 + r_{i,t}) \cdot w_{i,(t-1)} - c \qquad \text{with} \qquad r_t \sim \mathcal{N}(\mu, \sigma) \longrightarrow c = \mu \cdot w_0 \cdot k$$

c as an analogy to the reallocation term…



- cumulative advantage → non-ergodic dynamics → power law distributions

Many thanks for your attention!

Key sources:

Kapeller, Jakob and Steinerberger, Stefan (2025): Why are there so many power laws in economics? *Ifso working paper #50,* https://www.uni-due.de/imperia/md/content/soziooekonomie/ifsowp50_kapellersteinerberger2025.pdf

Dominy, Jonas (2024): Does Inequality Feed on Itself? Exploring the Explosive Dynamics of Wealth and Income. Master's Thesis, University of Duisburg-Essen.

Schulz, Jan and Weber, Jan David (2025): Power Laws in Socio-Economics. BERG Working Paper Series, No. 203, https://www.uni-bamberg.de/fileadmin/uni/fakultaeten/sowi_faecher/vwl/BERG/BERG_203.pdf